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Recent phenomenological models which posit extra spacelike dimensions in which (bulk) neutrinos are allowed to propagate are shown to have significant cosmological effects whenever the size of the largest extra dimension is $R \gtrsim 1\,\mathrm{fm}$ ($R^{-1} \lesssim 200\,\mathrm{MeV}$). Specifically, limits from the cosmic microwave background (CMB) anisotropies as measured by BOOMERanG, big bang nucleosynthesis, deuterium and ^{6}Li photoproduction, diffuse photon backgrounds, and structure formation/age considerations are shown to translate into broad constraints on bulk neutrino schemes. These present challenges for many recent light neutrino mass models invoking large volume toroidal compactifications as well as those involving densely spaced Kaluza-Klein (KK) modes. We discuss how these models would have to be modified to escape constraint and under what finely-tuned circumstances bulk neutrino KK tower states could constitute a new kind of dark matter. Future CMB observations (*e.g.*, MAP, Planck) may extend these constraints or discover signatures of bulk neutrino models with $R \sim 0.1\,\mathrm{fm}$.

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In this Letter we decsribe several cosmological constraints on models for neutrino mass which rely on bulk fermions propagating in compact extra spacelike dimensions. Extra spacetime dimensions, long provided under the ægis of Kaluza-Klein (KK) and superstring theories, have played an essential role in recent attempts to solve fundamental problems in particle physics [1,2]. In particular, in some theories invoking n compact extra spacelike dimensions, all Standard Model (SM) fields are localized on a three-dimensional surface (3-brane), but gravity experiences the full spacetime (bulk) [2]. This yields the relation $M_{\rm Pl}^2 = M_F^{n+2} V_n$ between the new fundamental $(4+n)$-dimensional reduced Planck scale M_F and the four-dimensional reduced Planck scale $M_{\rm Pl} = (4\pi G_N)^{-1/2}$, where V_n is the volume of the additional dimensions. If the volume of the internal space is sufficiently large, M_F can be much smaller than $M_{\rm Pl}$, giving rise to a low-scale theory of quantum gravity (*e.g.*, $V_2 \approx 5\times 10^{-5}\,\mathrm{mm}^2$ gives $M_F \sim 10\,\mathrm{TeV}$ for $n=2$).

In this framework, if M_F is sufficiently small, there is no longer a heavy mass scale available in the theory to suppress neutrino masses relative to other fermion masses via a seesaw or similar mechanism [3]. Several alternative, intrinisically higher-dimensional mechanisms have been developed [4], however, which can give neutrino masses and mixings consistent [5–7] with the solar, atmospheric, and accelerator neutrino experiments [8]. One widely used scheme posits the existence of SM-singlet fermions (neutrinos) which propagate in the bulk but couple via Yukawa interactions with the SM-doublet (active) neutrinos on our brane. This setup gives up to three light Dirac neutrino masses μ_i associated with the active neutrino flavors ν_e, ν_μ, and/or ν_τ. In addition, each bulk neutrino appears on our brane as a tower of massive KK modes (*i.e.*, sterile neutrinos), and the vacuum mixing angle between an active neutrino and a mode with mass $m_{\rm mode} \gg \mu_i$ is $\theta_{\rm mode} \simeq \sqrt{2}\mu_i/m_{\rm mode}$. The mass distribution of the modes depends on the geometry of the internal space. The simplest and most widely adopted geometry of the internal dimensions is that of an n-dimensional torus with radii R_j ($1 \le j \le n$), for which the mode masses are $m_{\mathbf{k}}^2 = k_1^2/R_1^2 + \cdots + k_n^2/R_n^2$, where, in bulk neutrino models, $\mathbf{k} = (k_1, \cdots, k_n)$ is an n-tuple of whole numbers, and where we assume that the bare masses of the bulk fermions are negligible. Using this framework, several authors have found non-standard solutions to the neutrino anomalies, under the assumption of toroidal compactification [5–7]. All of these solutions require $R_1^{-1} \lesssim 1\,\mathrm{eV}$ ($R_1 \gtrsim 0.1\,\mu\mathrm{m}$) for the largest dimension R_1, for otherwise they reduce to those for a standard Dirac neutrino mass [8,9]. In what follows, we show how these and non-toroidally compactified models with densely distributed KK modes affect standard cosmology through their incoherent production in the early universe and their subsequent decay.

The incoherent production of sterile-KK neutrinos of mass m_k in the early universe is a nonthermal process governed by a Boltzmann equation [10,11]

$$\frac{d}{dt} f_k = \Gamma(\nu_\alpha \to \nu_k) f_\alpha - \frac{m_k}{E}\frac{1}{\tau_k} f_k + \sum_{l>k} \mathrm{C}_{k,l}[f_l], \quad (1)$$

where $f_i = f_i(p,t)$ are momentum- and time-dependent distribution functions, and where α is an active neutrino label and k,l are a mode labels of a specific KK tower. We have specialized our discussion to the case in which $R = R_1$ is the radius of the largest extra dimension and all other dimensions are small enough to have no effect on sub-TeV neutrino physics, and we have ignored the flavor coupling of multiple towers [12]. Our results may easily be generalized to other cases. The first term in Eq. (1) is the conversion rate from active to sterile species and the second results from the decay of a mode with lifetime τ_k. The latter arises because singlet neutrinos which mix with active neutrinos can decay either to SM or bulk states. On the brane, the partial decay width of the process $\nu_k \to 3\nu$ is $\sin^2\theta_k G_F^2 m_k^5/192\pi^3 = G_F^2 m_k^3 \mu_i^2/96\pi^3$

and that of the radiative decay $\nu_k \to \nu\gamma$ is smaller by a factor $27\alpha/8\pi$ [13]. We have also included in our calculations the contributions to τ_k from visible and hadronic decays estimated from the partial decay widths of the Z^0-boson [14]. In the bulk, the k'-summed width of the process $\nu_k \to \nu_{k'} h_{k-k'}$ is $\sim m_k^4 R/12\pi M_{\rm Pl}^2$, where $h_{k-k'}$ is a KK graviton mode [15]. The last term in Eq. (1) represents the decay contribution of all higher modes $l > k$ into mode k, and $\mathrm{C}_{k,l}$ is the appropriate collision operator.

The conversion rate to KK modes is the product of half the interaction rate Γ of the neutrinos with the plasma and the average probability that an active neutrino scatters into mode k: $\Gamma(\nu_\alpha \to \nu_k) = \Gamma(\nu_\alpha \to \nu_k; p, t) = (\Gamma/2)\langle P(\nu_\alpha \to \nu_k)\rangle$. The probability depends on the amplitude of the matter mixing angle and the damping rate $D = \Gamma/2$ [16]:

$$\langle P(\nu_\alpha \to \nu_k)\rangle \simeq \frac{1}{2}\frac{\Delta_k^2 \sin^2 2\theta_k}{\Delta_k^2 \sin^2 2\theta_k + D^2 + (\Delta_k \cos 2\theta_k - V)^2}. \quad (2)$$

Here, $\Delta \simeq m_k^2/2p$, and $V = V^L + V^T$ is the full weak potential including lepton and thermal contributions. In this discussion, we assume a small lepton number of order the baryon number (so $V^L \ll V^T$), since a larger lepton number serves only to enhance sterile neutrino or anti-neutrino production. Our expression for the probability is derived from the standard physically well-motivated two-neutrino active-sterile matter mixing angle which includes the effects of quantum damping [16]. Our constraints depend on the deleterious effects of the relatively high modes, in which regime this formalism is identical to that derived from direct diagonalization of the tower mass matrix [6]. The contributions to the finite temperature potential V^T from the neutrino and charged lepton backgrounds of the same flavor are $(8\sqrt{2}G_F E_\nu/3m_{\rm Z}^2)(\rho_\nu + \rho_{\bar\nu})$ and $(8\sqrt{2}\,G_F E_\nu/3m_{\rm W}^2)(\rho_l + \rho_{\bar l})$, respectively [17]. This latter term must be included at high temperatures where e, μ, or τ leptons are populated [11].

Another important effect is the dilution of modes populated at $T \gg 100\,\mathrm{MeV}$. Disappearance of relativistic degrees of freedom manifests as heating of the plasma relative to the KK modes. We include this effect by following separately the complete time-temperature relations for the photons and modes.

We consider two classes of models: Class I with KK towers associated with all three active neutrinos; and Class II with no KK tower associated with the heaviest mass eigenvalue μ_3. In the first Class $\mu_3 > \sqrt{\delta m^2_{\rm SK}} \approx 0.057\,\mathrm{eV}$ [18], and in the second $\mu_{1,2} < 0.057\,\mathrm{eV}$. Overproduction of bulk graviton modes limits temperatures in these models to be less than the “normalcy” temperature $T_* \lesssim 10^{(6n-15)/(n+2)}\,\mathrm{MeV}(M_*/\,\mathrm{TeV})$ [19], where $M_*^{n+2} \equiv (2\pi)^n M_F^{n+2}$. Models falling into Class I correspond to large T_* ($\gtrsim T_{\rm electroweak} \sim 100\,\mathrm{GeV}$), while those

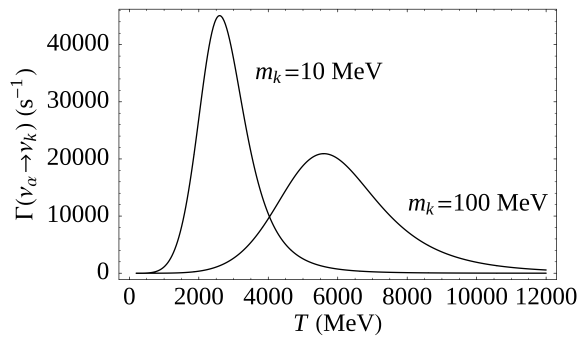

FIG. 1. $\Gamma(\nu_\alpha \to \nu_k)$ vs. T for $m_k = 10\,\mathrm{MeV}$, $100\,\mathrm{MeV}$ and $\mu_i = 3\,\mathrm{eV}$.

in Class II, because they invoke large R, must necessarily have very low T_* ($\lesssim$ GeV).

For each of these classes we have solved numerically Eq. (1) with $\mathrm{C}_{k,l} = 0$ for the population of the N lowest modes, with fully self-consistent temperature evolution of all relevant species. The height N of the tower is the highest mode populated near the appropriate T_*. We have conservatively incorporated the effects of decays in the bulk by assuming the decay products' mass-energy negligibly affects the dynamics of the universe. The population rates $\Gamma(\nu_\alpha \to \nu_k)$ for two particular modes are shown in Fig. 1. Although the peak value of $\Gamma(\nu_\alpha \to \nu_k)$ falls with increasing mode mass, its temperature width increases concomitantly. Therefore, the integrated population rate (and energy density) of the mode turns out to be *independent* of the mode number at the decoupling temperature $T_{\nu\rm dec} \sim 1\,\mathrm{MeV}$ of the active neutrinos, despite the dependence of the mixing angle on mode mass.

This result depends on the mode being non-relativistic and not having appreciably decayed [10,11]. Under these assumptions, and with some simplifications, we can use Eq. (9) in Ref. [10] to obtain an analytic estimate $N_{\nu_k}(\mathrm{BBN}) \sim 10^{-3}\ (\mu_i/1\,\mathrm{eV})^2 (g_*^{\rm f}/g_{*k}^{\rm p})$ for the energy density at $T_{\nu\rm dec}$ in a single mode k relative to that in an active neutrino species. The ratio $(g_*^{\rm f}/g_{*k}^{\rm p})$ approximates dilution effects. The statistical weight in relativistic particles in the plasma at $T_{\nu\rm dec}$ is $g_*^{\rm f}$, and is $g_{*k}^{\rm p}$ at the epoch of maximal production of mode k. (Roughly, this maximal production epoch is related to mode mass as $T_{\rm max} \simeq 133\,\mathrm{MeV}(m_k/1\,\mathrm{keV})^{1/3}$ [10].) Our numerical calculations follow in detail the simultaneous production, dilution, and decay of all relevant modes of various energies, giving a $N_{\nu_k}(\mathrm{BBN})$ dependence on k which is flat modulo the effects of dilution and decay.

Population of KK modes in the early universe leads to a number of unacceptable effects that provide for compelling constraints. Our calculated cosmological constraints differ from those in Ref. [6], but they complement the supernova limits claimed in Refs. [6,7].

Class I model constraints are given in Fig. 2. The total effective number $N_\nu(\mathrm{BBN})$ of neutrino flavors at the BBN epoch must be less than that of 4 active neutrino species, since otherwise the predicted and observed abundances

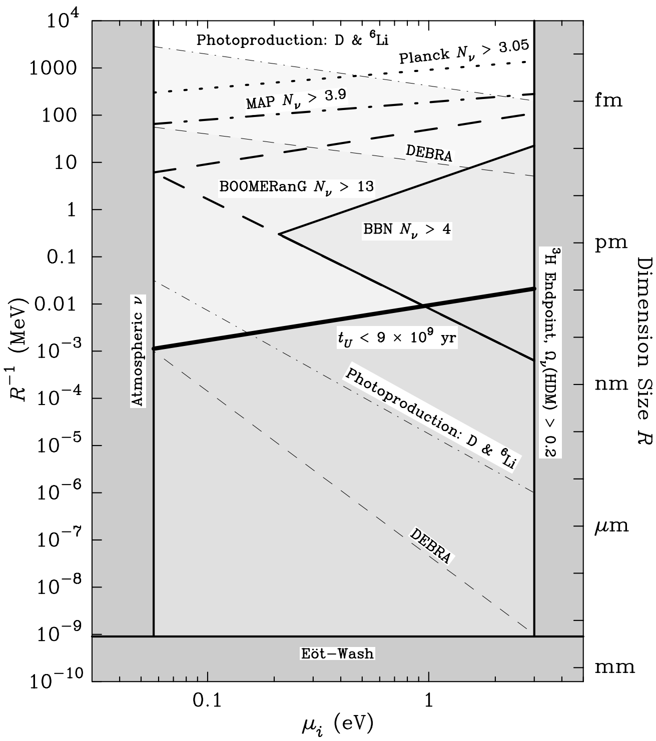

FIG. 2. Cosmological constraints on Class I bulk neutrinos. Photoproduction and DEBRA constraints lie between the dot-dashed and short-dashed lines, respectively. The CMB constraints lie below the labeled BOOMERanG, MAP, and Planck lines and above the long-dashed line. BBN constains the region between the light solid lines. Parameters must lie above the heavy solid line to be consistent with the inferred age of the universe. The vertical lines arise from the neutrino oscillation and ^{3}H endpoint limits on neutrino masses [8,9,14,18]. Also shown is the Eöt-Wash limit on the size of large extra dimensions [20].

of the light elements are discordant [21]. Since the active flavors contribute $N_{\nu_\alpha} = 3$, we require the KK tower contribution $\sum N_{\nu_k}$(BBN) to be less than that of a single active neutrino flavor. Photoproduction of deuterium (D) and ^{6}Li due to decay of modes after big bang nucleosynthesis (BBN) [23] gives another constraint. Energetic cascades dissociate ^{4}He into excessive amounts of D, which is bounded observationally [24]. The increase in energy density in relativistic particles due to mode decay prior to cosmic microwave background (CMB) decoupling can lead to suppression of the second CMB acoustic peak. The current limit is that the effective number of neutrino flavors at decoupling is $N_\nu(\mathrm{CMB}) < 13$ at 95% certainty [25]. Measurements to higher multipole moments by the MAP (reaching $N_\nu(\mathrm{CMB}) \simeq 3.9$) and Planck (reaching $N_\nu(\mathrm{CMB}) \simeq 3.05$) surveys will be able to further limit the relativistic energy present at decoupling [26], or perhaps flag the fossil relativistic energy of bulk modes at $R \sim 0.1$ fm. The increase in energy density due to mode decays was found by summing the energy injected between the neutrino and photon decoupling epochs. Another significant constraint comes from the current limits

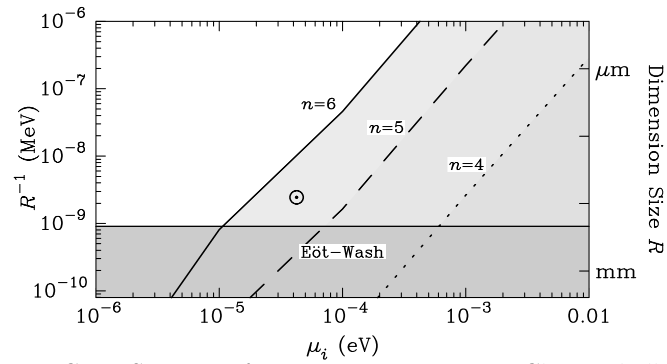

FIG. 3. Structure formation constraints on Class II bulk neutrino models with numbers of extra dimensions $n = 6, 5, 4$. For example, the solar neutrino solutions of Dvali & Smirnov and Caldwell, Mohapatra, & Yellin [5] lie near $\odot$.

on diffuse extra-galactic backround radiation (DEBRA) due to radiative decays of sterile neutrinos occuring between CMB decoupling and today. The photon background so produced must have a total flux per unit solid angle $d\mathcal{F}/d\Omega \lesssim (1\,\mathrm{MeV}/E)\ \mathrm{cm}^{-2}\mathrm{sr}^{-1}\mathrm{s}^{-1}$ [27–29]. The expansion age of the universe, $t_U > 9 \times 10^9$ yr, also provides a constraint on the energy density in KK modes. We have found that that the constraints from distortion of the CMB spectrum and the signals in the solar neutrino experiments from mode decays are weaker than the constraints above [27,29,30]. Note that the arguments above do not depend on the detailed mode structure but rather on the existence of a high density of modes.

Class II model effects are shown in Fig. 3. Though high-lying modes in these models are absent owing to low T_*, there may remain enough energy density in low mass modes to comprise an appreciable hot dark matter component. Structure formation considerations [22] suggest that a hot component cannot contribute $\Omega^{\mathrm{HDM}} > 0.1$. Contours of $\Omega^{\mathrm{HDM}}_{\nu_s} = 0.1$ are shown in Fig. 3 for these models with $n = 6, 5, 4$ extra dimensions. Note that some recent models for solar neutrino oscillations fall in a parameter range which could give an appreciable $\Omega^{\mathrm{HDM}}_{\nu_s}$. Whether this can constitute a true constraint depends on the precise relation between T_* and M_* in these models. At present all Class II models [5–7] can escape elimination by invoking low T_* ($\lesssim 20$ MeV).

The spectrum of low-lying modes in Class II models could give a viable dark matter candidate if T_* is low. Low T_* results in suppressed production of high-mass modes that provide the closure, BBN, and decay constraints. For low enough μ_i, all modes produced below T_* survive until today and escape decay constraints. The possibility of a realistic dark matter component from the KK modes is finely tuned. For instance, for $R^{-1} = 4 \times 10^{-8}$ MeV, and $\mu_i = 10^{-5}$ eV, $\Omega_{\nu_s} \sim 0.1$ for $T_* \sim 1$ GeV, but $\Omega_{\nu_s} \sim 0.2$ for $T_* \gtrsim 1.3$ GeV. Albeit finely tuned, the latter case, for which $\Omega^{\mathrm{HDM}}_{\nu_s} < 0.1$, is an interesting dark matter candidate, comprising a mixture of hot, warm ($\sim$ keV), and cold ($\sim$ MeV) components.

Modifications to Class I and II models may allow circumvention of our constraints. First, a stronger dependence of the mixing angle on $\mu_i/m_{\rm mode}$ would ensure that the population of the modes would instead fall with increasing mass, implying that their cosmological effects are benign. Second, an alternate dependence of the lifetime of a mode on its mass and μ_i could eliminate some or all of the constraints. Third, there could exist multiple additional (possibly fat) branes in the bulk, devoid of energy density and parallel to our own, onto which modes decay preferentially [19]. Fourth, if the re-heating temperature T_* of inflation is $T_{\nu{\rm dec}}$, no KK modes will be populated, unless there is a large lepton number. Fifth, the internal dimensions need not be toroidally compactified [31]. An internal space which has a KK mode decomposition with a sufficiently low density of modes (or a *gap*) between the lowest and highest modes which are significantly populated could evade cosmological constraints. Sixth, bad effects of higher μ_i can be removed by restricting the KK towers to those built on low μ_i ($< 10^{-4}\,$eV) active neutrinos — this is what is (or should be) done in Class II models to avoid constraint. However, models which make use of all three towers are in some sense the most "natural," albeit the most severely constrained.

Ultimately, our cosmological considerations may help to narrow the otherwise prodigious range of parameters discussed by modelers to date.

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[1] See, *e.g.*, E. Witten, hep-ph/0002297, and references therein.

[2] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. **B429**, 263 (1998).

[3] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D.Z. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, in *Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe*, edited by A. Sawada and A. Sugamoto, K.E.K. preprint 79-18 (1979); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[4] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, hep-ph/9811448; K.R. Dienes, E. Dudas, and T. Gherghetta, Nucl. Phys. **B557**, 25 (1999); K.R. Dienes and I. Sarcevic, hep-ph/0008144.

[5] G. Dvali and A.Yu. Smirnov, Nucl. Phys. **B563**, 63 (1999); R.N. Mohapatra, S. Nandi, and A. Pérez-Lorenzana, Phys. Lett. **B466**, 115 (1999); R.N. Mohapatra and A. Pérez-Lorenzana, Nucl. Phys. **B576**, 466 (2000); R. Barbieri, P. Creminelli, and A. Strumia, *ibid.* **B585**, 28 (2000); R.N. Mohapatra and A. Pérez-Lorenzana, hep-ph/0006278; N. Cosme *et al.*, hep-ph/0010192; D.O. Caldwell, R.N. Mohapatra, and S. J. Yellin, hep-ph/0010353.

[6] R. Barbieri, P. Creminelli, and A. Strumia, Nucl. Phys. **B585**, 28 (2000).

[7] A. Lukas, P. Ramond, A. Romanino, and G.G. Ross, hep-ph/0008049.

[8] For a review, see, *e.g.*, S.M. Bilenky, C. Giunti, and W. Grimus, Prog. Part. Nucl. Phys. **43**, 1 (1999).

[9] J.N. Bahcall, P. Krastev, A.Yu. Smirnov, Phys. Rev. D **58**, 096016 (1998); M.C. Gonzalez-Garcia, M. Maltoni, C. Peña-Garay, and J.W.F. Valle, hep-ph/0009350.

[10] S. Dodelson and L.M. Widrow, Phys. Rev. Lett. **72**, 17 (1994).

[11] K. Abazajian, G.M. Fuller, and M. Patel, in preperation.

[12] Though such mixing is an essential feature of many models, it has little effect on our estimates of mode population, since unitarity ensures that the three active species contribute to three full KK towers.

[13] V. Barger, R.J.N. Phillips, and S. Sarkar, Nucl. Phys. **B352**, 365 (1995); **B356**, 617 (1995).

[14] D.E. Groom *et al.*, Eur. Phys. J. **C15**, 1 (2000).

[15] R.N. Mohapatra, A. Pérez-Lorenzana, and C.A. de S. Pires, Phys. Rev. D **62**, 105030 (2000).

[16] R. Foot, M.J. Thomson, and R.R. Volkas, Phys. Rev. D **53**, 5349 (1996); R.R. Volkas and Y.Y.Y. Wong, hep-ph/0007185.

[17] D. Nötzold and G. Raffelt, Nucl. Phys. **B307**, 924 (1988).

[18] H. Sobel, talk given at the XIX International Physics & Astrophysics, Sudbury, Canada, June 16-12, 2000.

[19] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Rev. D **59**, 086004 (1999).

[20] C.D Hoyle *et al.*, hep-ph/0011014.

[21] G. Steigman, D.N. Schramm, and J.R. Gunn, Phys. Lett. **B66**, 202 (1977); E. Lisi, S. Sarkar, and F.L. Villante, Phys. Rev. D **59**, 123520 (1999); K. Abazajian, G.M. Fuller, and X. Shi, *ibid.* **62**, 093003 (2000).

[22] J.R. Primack and M.A.K. Gross, in *Current Aspects of Neutrino Physics*, edited by D.O. Caldwell, (Springer, Berlin Heidelberg, 2000), astro-ph/0007165; E. Gawiser, astro-ph/0005475.

[23] See K. Jedamzik, Phys. Rev. Lett. **84**, 3248 (2000), and references therein.

[24] S. Burles and D. Tytler, Astrophys. J. **499**, 699 (1998); **507**, 732 (1998).

[25] S. Hannestad, Phys. Rev. Lett., in press, astro-ph/0005018.

[26] R.E. Lopez, S. Dodelson, A. Heckler, and M.S. Turner, Phys. Rev. Lett. **82**, 3952 (1999).

[27] D.A. Dicus, E.W. Kolb, and V.L. Teplitz, Astrophys. J. **221**, 327 (1978).

[28] M.T. Ressell and M.S. Turner, Comments Astrophys. **14**, 323 (1989).

[29] E.W. Kolb and M.S. Turner, *The Early Universe* (Addison-Wesley, Reading, Massachusetts, 1993).

[30] J.N. Bahcall, *Neutrino Astrophysics* (Cambridge University Press, New York, 1989).

[31] N. Kaloper, J. March-Russell, G.D. Starkman, and M. Trodden, Phys. Rev. Lett. **85**, 928 (2000).